(10 pts) 1. Consider the following set

\[ A = (0, 1) \cup \{-n, n = 1, 2, \ldots\} \]

Determine if the set is open, closed, has isolated points or not. Provide explanation for each answer. Find all the limit points of the set. Find the closure of \( A \).

(10 pts) 2. Verify, using the definition of convergence of a sequence, that the following sequence converges to the proposed limit

\[ \lim_{n \to \infty} \frac{\sin(n^2)}{\sqrt{n}} = 0. \]

(10 pts) 3. Provide a proof for each of the statement below or explain why the request is impossible.

(a) A sequence of continuous functions \( f_n \) converging uniformly to the function

\[ f(x) = \begin{cases} 
0 & x \geq 0 \\
1 & x < 0 
\end{cases} \]

on the interval \([-1, 1]\).

(b) An absolutely convergent series \( \sum a_n \) implies that \( \sum \frac{a_n}{n} \) is also convergent.

(c) There exists a power series \( \sum a_n x^n \) converging for all \( x \geq 0 \) but divergent at \( x = -1 \).

(d) If a power series \( \sum a_n x^n \) with positive coefficients \( a_n \geq 0 \) converges conditionally at \( x = -1 \), then it diverges at \( x = 1 \).

(10 pts) 4. Derive the Taylor series of \( \cos(x) \). Show that the series converges to \( \cos(x) \) at every point.

(10 pts) 5. Let \( g \) be a differentiable function function on interval \([0, 3]\). Suppose we know that \( g(0) = 1 \), \( g(2) = -1 \), \( g(3) = 2 \).

(a) Show that there exists a point \( a \in [0, 3] \) such that \( g(a) = 0 \);

(b) Show that there exists a point \( b \in [0, 3] \) such that \( g'(b) = 1/3 \);

(c) Show that there exists a point \( c \in [0, 3] \) such that \( g'(c) = -1 \);

(10 pts) 6. Give the definition of a Riemann integrable function on \([a, b]\). Show that \( f(x) = x \) is Riemann integrable on \([0, 1]\) and find its integral.

(10 pts) 7. Let \( F(x) = \int_0^x f(t)dt \), and assume that \( f \) is a Riemann integrable function with \( f(t) > 2 \) for all \( t \). Show that \( F(1) > 2 \), and prove that \( F \) attains value 1 on the interval \((0, 1)\).