Midterm Exam 1
Duration: 1 hour
Total: 50 points

(10 pts) 1. (a) State the definition of a convergent series.
   (b) State the definition of a Cauchy sequence.

(10 pts) 2. Show that $\sqrt{3}$ is not a rational number.

(10 pts) 3. Prove using the definition of a limit that
   $$\lim_{n \to \infty} \frac{1}{\sqrt{n} + 3} = 0.$$ 

(10 pts) 4. Give an example to the statements below or if such a request is impossible explain why.
   (a) A convergent sequence $(a_n)$ which contains two subsequences converging to 0 and 1, respectively.
   (b) A divergent sequence containing a Cauchy subsequence.
   (c) A divergent sequence $(a_n)$ and another divergent sequence $(b_n)$ such that $(a_n - b_n)$ is convergent.

(10 pts) 5. A sequence is given by
   $$\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \ldots$$
   Prove that the sequence converges, and find its limit.